

A-2 Strengthening Your Math Instruction with Research-Affirmed Strategies (Gr. 3-6)



STEVE LEINWAND

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PUBLISHED WORKS:

Teacher Resources:

Accessible Math: 10 Instructional Shifts that Raise Student Achievement

Sensible Mathematics: A Guide for School Leaders

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Strengthening Your Grades 3-6 Math Instruction with Research-Affirmed Strategies

SDE Summer Conference
July, 2011

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What a great time to be convening as teachers of math!

- Common Core State Standards
- Quality K-8 materials
- \$5 billion with a STEM RttT tie-breaker
- A president who believes in science and data
- The beginning of the end of Algebra II
- A long overdue understanding that it's instruction, stupid!

2

Logic model or theory of action

(aka where we sit in the food chain)

Economic security and social well-being

↑ ↑ ↑

Innovation and productivity

↑ ↑ ↑

Human capital and equity of opportunity

↑ ↑ ↑

High quality education

(literacy, MATH, science)

↑ ↑ ↑

Daily classroom math instruction

3

Automaticity

- Starting the car
- Withdrawing cash from an ATM
- My clicker

4

My Goal Today

Engage you in thinking about (and then being willing and able to act on) the issues of what we teach, how we teach, and how much they learn by:

- validating your concerns,
- examining standard operating procedures,
- giving you some tools and ideas for making math more accessible to our students,
- empowering you to collectively take risks.

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My content agenda

- Part 1: Introductory Thoughts and Issues
- Part 2: It's Instruction, Silly
- Part 3: Tying Things Together
- Part 4: Final Thoughts on Moving Forward

6

My Process Agenda (modeling good instruction)

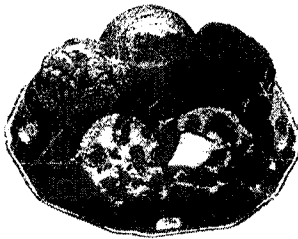
- **Inform** (lots of ideas and food for thought)
- **Engage** (focused individual and group tasks)
- **Stimulate** (excite your sense of professionalism)
- **Challenge** (urge you to move from words to action)

7

Ready?

8

Breakfast or dessert?



9



Free
3 Muffins when you
buy 3 at the regular
1/2 dozen price

Available only at participating Dunkin' Donuts® shops.
One coupon per customer per visit. May not be
combined with any other offers or promotions.
Offer must remain open. Taxes not included.
1 offer
1 offer
DUNKIN' DONUTS
It's more the way.

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NCTM Standards

Process Standards	Content Standards
<ul style="list-style-type: none"> • Problem Solving • Reasoning and Proof • Communication • Connections • Representations 	<ul style="list-style-type: none"> • Number • Measurement • Geometry • Algebra • Data

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All the standards rolled up into one:

- **Problem Solving:** What is this? What's that white thing?
- **Communication:** Tell the person sitting next to you.
- **Reasoning:** How do you know?
- **Connections:** A real rip-off ad.
- **Representations:** A picture

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But look at what else this example shows us:
Consider how we teach reading:
JANE WENT TO THE STORE.

- Who went to the store?
- Where did Jane go?
- Why do you think Jane went to the store?
- Do you think it made sense for Jane to go to the store?

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**Now consider mathematics:
TAKE OUT YOUR HOMEWORK.**

- #1 19
- #2 37.5
- #3 185

(No why? No how do you know? No who has a different answer?)

14

Strategy #1

**Adapt from what we know
about reading
(incorporate literal, inferential,
and evaluative
comprehension to develop
stronger neural connections)**

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The bottom line

**Good math teaching
BEGINS with an answer
(often a wrong answer)**

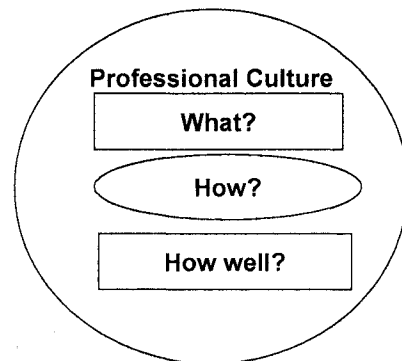
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Part 1

Introductory Thoughts and Issues

**(glimpses at the what, why and
how of what we do)**

17



18

There is no valid psychological or logical reason to limit students of lesser academic ability or aptitude to practice with paper and pencil procedures.

On the contrary, there is ample evidence to suggest that such an approach is often counter-productive, resulting in little improvement in procedural skills and increasingly negative attitudes.

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from Everybody Counts

Virtually all young children like mathematics. They do mathematics naturally, discovering patterns and making conjectures based on observation. Natural curiosity is a powerful teacher, especially for mathematics....

20

Unfortunately, as children become socialized by school and society, they begin to view mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed, and memory. Their view of mathematics shifts gradually from enthusiasm to apprehension, from confidence to fear. Eventually, most students leave mathematics under duress, convinced that only geniuses can learn it.

21

Accuracy, Speed and Memory

Tell the person sitting next to you what is the formula for the volume of a sphere.

$$V = \frac{4}{3} \pi r^3$$



4/3 ? r? 3? π?

22

Sucking intelligence out...

Late one night a shepherd was guarding his flock of 20 sheep when all of a sudden 4 wolves came over the hill.

Boys and girls, how old was the shepherd?

23

So...the problem is:

If we continue to do what we've always done....

We'll continue to get what we've always gotten.

24


Practice Plus

Key Skill: Subtraction, page 27
Subtract

1. $\begin{array}{r} 32 \\ - 17 \\ \hline \end{array}$ $\begin{array}{r} 48 \\ - 23 \\ \hline \end{array}$ $\begin{array}{r} 86 \\ - 65 \\ \hline \end{array}$ $\begin{array}{r} 54 \\ - 31 \\ \hline \end{array}$ $\begin{array}{r} 69 \\ - 19 \\ \hline \end{array}$

2. $\begin{array}{r} 77 \\ - 46 \\ \hline \end{array}$ $\begin{array}{r} 51 \\ - 27 \\ \hline \end{array}$ $\begin{array}{r} 98 \\ - 25 \\ \hline \end{array}$ $\begin{array}{r} 66 \\ - 33 \\ \hline \end{array}$ $\begin{array}{r} 40 \\ - 16 \\ \hline \end{array}$ $\begin{array}{r} 83 \\ - 50 \\ \hline \end{array}$

3. $\begin{array}{r} 53 \\ - 24 \\ \hline \end{array}$ $\begin{array}{r} 34 \\ - 8 \\ \hline \end{array}$ $\begin{array}{r} 74 \\ - 26 \\ \hline \end{array}$ $\begin{array}{r} 95 \\ - 48 \\ \hline \end{array}$ $\begin{array}{r} 57 \\ - 19 \\ \hline \end{array}$ $\begin{array}{r} 24 \\ - 10 \\ \hline \end{array}$



Key Skill: Subtraction Money, page 28
Subtract

1. $\begin{array}{r} 27c \\ - 10c \\ \hline \end{array}$ $\begin{array}{r} 31c \\ - 26c \\ \hline \end{array}$ $\begin{array}{r} 44c \\ - 27c \\ \hline \end{array}$ $\begin{array}{r} 53c \\ - 29c \\ \hline \end{array}$ $\begin{array}{r} 97c \\ - 59c \\ \hline \end{array}$ $\begin{array}{r} 80c \\ - 41c \\ \hline \end{array}$

2. $\begin{array}{r} 49c \\ - 23c \\ \hline \end{array}$ $\begin{array}{r} 71c \\ - 59c \\ \hline \end{array}$ $\begin{array}{r} 89c \\ - 39c \\ \hline \end{array}$ $\begin{array}{r} 56c \\ - 37c \\ \hline \end{array}$ $\begin{array}{r} 65c \\ - 48c \\ \hline \end{array}$ $\begin{array}{r} 92c \\ - 13c \\ \hline \end{array}$

3. $\begin{array}{r} 38c \\ - 25c \\ \hline \end{array}$ $\begin{array}{r} 52c \\ - 36c \\ \hline \end{array}$ $\begin{array}{r} 84c \\ - 68c \\ \hline \end{array}$ $\begin{array}{r} 67c \\ - 43c \\ \hline \end{array}$ $\begin{array}{r} 75c \\ - 27c \\ \hline \end{array}$ $\begin{array}{r} 99c \\ - 11c \\ \hline \end{array}$

One seven-year-old student's
viewpoint of life at 100

If I were 100 years old
~~~~~  
If I were 100 years old,  
I would go to a nursing  
home. I would stay  
there until I was dead.  
By the time I was 100,  
I would know regrouping  
with subtraction and  
then I would die happy.

26

Alg 2b Eq Exam Do not write on this exam

25

Perform the indicated operation and simplify.

1.  $\frac{9x^2}{12x^2+8} \cdot \frac{1}{x^2+1} = \frac{3}{4(x^2+1)}$

2.  $\frac{x^2-x+2}{x^2(x^2+1)} \cdot \frac{x}{x+2} = \frac{x}{x^2(x+1)}$

3.  $\frac{3x^2}{x^2+1} \cdot \frac{2x^2+1}{x^2+1} = \frac{6x^4+3x^2}{x^4+2x^2+1}$

4.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

5.  $\frac{3x^2+5x-3}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(3x+1)(x-3)(x-2)}{(x^2+1)(x+2)}$

6.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

7.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

8.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

9.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

10.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

11.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

12.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

13.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

14.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

15.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

16.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

17.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

18.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

19.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

20.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

21.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

22.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

23.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

24.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

25.  $\frac{x^2+3x+2}{x^2+1} \cdot \frac{x-2}{x+2} = \frac{(x+1)(x+2)(x-2)}{(x^2+1)(x+2)} = \frac{(x+1)(x-2)}{x^2+1}$

[illegible]

10.  $\frac{1}{x} = \frac{1}{x} \cdot \frac{x}{x} = \frac{x}{x^2} = \frac{1 \cdot x^1}{x^2} = \frac{1 \cdot x^{1-2}}{x^2} = \frac{1 \cdot x^{-1}}{x^2} = \frac{1}{x^2}$   
 11.  $\frac{12x^2 + 10x - 3}{x^2} = \frac{12x^2}{x^2} + \frac{10x}{x^2} - \frac{3}{x^2} = 12 + \frac{10}{x} - \frac{3}{x^2}$   
 12.  $\sqrt{16x^2 + 24x + 9} = \sqrt{(4x+3)^2} = 4x+3$   
 13.  $\frac{1}{x^2} = \frac{1}{x^2} \cdot \frac{x}{x} = \frac{x}{x^3} = \frac{1 \cdot x^1}{x^3} = \frac{1 \cdot x^{1-3}}{x^3} = \frac{1 \cdot x^{-2}}{x^3} = \frac{1}{x^2}$   
 14.  $\frac{1}{x^2} = \frac{1}{x^2} \cdot \frac{x}{x} = \frac{x}{x^3} = \frac{1 \cdot x^1}{x^3} = \frac{1 \cdot x^{1-3}}{x^3} = \frac{1 \cdot x^{-2}}{x^3} = \frac{1}{x^2}$   
 15.  $\frac{1}{x^2} = \frac{1}{x^2} \cdot \frac{x}{x} = \frac{x}{x^3} = \frac{1 \cdot x^1}{x^3} = \frac{1 \cdot x^{1-3}}{x^3} = \frac{1 \cdot x^{-2}}{x^3} = \frac{1}{x^2}$   
 16.  $\frac{1}{x^2} = \frac{1}{x^2} \cdot \frac{x}{x} = \frac{x}{x^3} = \frac{1 \cdot x^1}{x^3} = \frac{1 \cdot x^{1-3}}{x^3} = \frac{1 \cdot x^{-2}}{x^3} = \frac{1}{x^2}$   
 17.  $\frac{1}{x^2} = \frac{1}{x^2} \cdot \frac{x}{x} = \frac{x}{x^3} = \frac{1 \cdot x^1}{x^3} = \frac{1 \cdot x^{1-3}}{x^3} = \frac{1 \cdot x^{-2}}{x^3} = \frac{1}{x^2}$   
 18.  $\frac{1}{x^2} = \frac{1}{x^2} \cdot \frac{x}{x} = \frac{x}{x^3} = \frac{1 \cdot x^1}{x^3} = \frac{1 \cdot x^{1-3}}{x^3} = \frac{1 \cdot x^{-2}}{x^3} = \frac{1}{x^2}$   
 19.  $\frac{1}{x^2} = \frac{1}{x^2} \cdot \frac{x}{x} = \frac{x}{x^3} = \frac{1 \cdot x^1}{x^3} = \frac{1 \cdot x^{1-3}}{x^3} = \frac{1 \cdot x^{-2}}{x^3} = \frac{1}{x^2}$   
 20.  $\frac{1}{x^2} = \frac{1}{x^2} \cdot \frac{x}{x} = \frac{x}{x^3} = \frac{1 \cdot x^1}{x^3} = \frac{1 \cdot x^{1-3}}{x^3} = \frac{1 \cdot x^{-2}}{x^3} = \frac{1}{x^2}$

**Algebra:**  
**The intense study of the  
last three letters of the  
alphabet**

## So what have we gotten?

- Mountains of math anxiety
- Tons of mathematical illiteracy
- Mediocre test scores
- HS programs that barely work for more than half of the kids
- Gobs of remediation and intervention
- A slew of criticism

Not a pretty picture!

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## If however.....

**What we've always done is no longer acceptable, then...**

**We have no choice but to change some of what we do and some of how we do it.**

32

## And thus some perspectives:

- We're being asked to do what has never been done before:  
Make math work for nearly ALL kids.
- We're therefore being asked to teach in distinctly different ways:  
Because there is no other way to serve a much broader proportion of students.

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"The kind of learning that will be required of teachers has been described as *transformative* (involving sweeping changes in deeply held beliefs, knowledge, and habits of practice) as opposed to *additive* (involving the addition of new skills to an existing repertoire). Teachers of mathematics cannot successfully develop their students' reasoning and communication skills in ways called for by the new reforms simply by using manipulatives in their classrooms, by putting four students together at a table, or by asking a few additional open-ended questions....."

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Rather, they must thoroughly overhaul their thinking about what it means to know and understand mathematics, the kinds of tasks in which their students should be engaged, and finally, their own role in the classroom."

NCTM – Practice-Based Professional Development for Teachers of Mathematics

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## Not convinced?

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**SUBWAY** THE NEW YORK CITY SUBWAY AUTHORITY  
**EMPLOYMENT APPLICATION**  
 PERSONAL INFORMATION: Name, Address, City, State, Zip, Phone, Social Security Number, Date of Birth, Sex, Race, Religion, Marital Status, Number of Children, Date of Last Employment, Reason for Leaving, Date of Last Employment, Reason for Leaving, Date of Last Employment, Reason for Leaving.

**EDUCATION:** School, Grade, Date of Graduation, Date of Last Employment, Reason for Leaving, Date of Last Employment, Reason for Leaving, Date of Last Employment, Reason for Leaving.

**EMPLOYMENT HISTORY:** Employer, Address, City, State, Zip, Phone, Date of Employment, Reason for Leaving, Date of Last Employment, Reason for Leaving, Date of Last Employment, Reason for Leaving.

**REFERENCES:** Name, Address, City, State, Zip, Phone, Date of Reference, Reason for Reference, Date of Last Reference, Reason for Last Reference, Date of Last Reference, Reason for Last Reference.

**DECLARATION:** I hereby declare that the information furnished by me is true and correct to the best of my knowledge and belief. I understand that any false statement made by me may result in my being disqualified for employment.

**SIGNATURE:** Name, Address, City, State, Zip, Phone, Date of Signature, Reason for Signature, Date of Last Signature, Reason for Last Signature, Date of Last Signature, Reason for Last Signature.

37

**EMPLOYMENT TEST**  
 (For Candidates Only)

**PART I:**

|    |       |
|----|-------|
| 80 | 10.00 |
| 75 | 4.55  |
| 50 | 3.00  |
| 25 | 2.75  |
| 10 | 2.50  |
| 5  | 2.25  |
| 2  | 2.00  |
| 1  | 1.75  |
| 0  | 1.50  |
| 0  | 1.25  |
| 0  | 1.00  |
| 0  | 0.75  |
| 0  | 0.50  |
| 0  | 0.25  |
| 0  | 0.00  |

**PART II:**

1. A customer complains that he was short charged by you receiving only 10¢ change from \$1.00. What would you do?

2. Return to your customer using proper technique as a Subway Sandwich Shop employee?

3. What do you consider to be the most important qualities in a Subway Sandwich Shop employee?

4. You are working alone and your shift is due to be over at 4 P.M. The individual who is scheduled to begin working at 4 P.M. does not show up. What would you do?

**DECLARATION:** I hereby declare that the information furnished by me is true and correct to the best of my knowledge and belief. I understand that any false statement made by me may result in my being disqualified for employment.

**SIGNATURE:** Name, Address, City, State, Zip, Phone, Date of Signature, Reason for Signature, Date of Last Signature, Reason for Last Signature, Date of Last Signature, Reason for Last Signature.

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Envision the last test you gave your students.

Compare your test with the Subway Employment Test.



39

Let's see if we can be hired.

40

10.00  
 - 4.59

41

If the customer's order came to \$6.22 and he gave you \$20.25, what is the change?

42



A customer complained that he was short changed by you, receiving only 13¢ from his \$2.00 instead of 31¢. What would you do?

43

Yes.

A lot to think about.

But if you think everything is hunky-dory, you're not going to change.

44

## Part 2

It's instruction, silly!

45

## So it's instruction, silly!

Research, classroom observations and common sense provide a great deal of guidance about instructional practices that make significant differences in student achievement. These practices can be found in high-performing classrooms and schools at all levels and all across the country. **Effective teachers make the question "Why?" a classroom mantra to support a culture of reasoning and justification. Teachers incorporate daily, cumulative review of skills and concepts into instruction. Lessons are deliberately planned and skillfully employ alternative approaches and multiple representations—including pictures and concrete materials—as part of explanations and answers. Teachers rely on relevant contexts to engage their students' interest and use questions to stimulate thinking and to create language-rich mathematics classrooms.**

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Accordingly:

**Some Practical,  
Research-Affirmed  
Strategies  
for  
Raising Student  
Achievement Through  
Better Instruction**

47

**My message today is  
simple:**

**We know what works!**

- K-1
- Reading
- Gifted
- Active classes
- Questioning classes
- Thinking classes

48

**Our job is to extract from these places and experiences specific strategies that can be employed broadly and regularly.**

49

**So let's play!**

50

### **Number from 1 to 6**

1. What is  $6 \times 7$ ?
2. What number is 1000 less than 294?
3. About how much is 32¢ and 29¢?
4. What is  $\frac{1}{10}$  of 450?
5. Draw a picture of  $1\frac{2}{3}$
6. About how much do I weight in kg?

51

### **Number from 1 to 6**

1. How much bigger is 9 than 5?
2. What number is the same as 5 tens and 7 ones?
3. What number is 10 less than 83?
4. Draw a four-sided figure and all of its diagonals.
5. About how long is this pen in centimeters?

52

### **Strategy #2**

**Incorporate on-going cumulative review into instruction every day.**

53

### **Implementing Strategy #2**

Almost no one masters something new after one or two lessons and one or two homework assignments. That is why one of the most effective strategies for fostering mastery and retention of critical skills is daily, cumulative review at the beginning of every lesson.

54

### On the way to school:

- A term of the day
- A picture of the day
- An estimate of the day
- A skill of the day
- A graph of the day
- A word problem of the day

55

**Great  
Take a deep breath!**

56

**Tell me what you see.**

**73  
63**

57

**Tell me what you see.**

**2 1/4**

58

### Strategy #3

**Create a language rich  
classroom.**

(Vocabulary, terms, answers,  
explanations)

59

### Implementing Strategy #3

Like all languages, mathematics must be encountered orally and in writing. Like all vocabulary, mathematical terms must be used again and again in context and linked to more familiar words until they become internalized.

Sum = both  
Area = covering  
Perimeter = border  
Circumference = a belt

Difference – bigger than  
Quotient = sharing  
Mg = grain of sand  
Surface area = skin

60

**Ready, set, picture.....**

**“three quarters”**

61

**Why does this make a difference?**

**Consider the different ways of thinking about the same mathematics:**

- $2\frac{1}{2} + 1\frac{3}{4}$
- $\$2.50 + \$1.75$
- $2\frac{1}{2}'' + 1\frac{3}{4}''$

62

**Ready, set, picture.....**

**20 centimeters**

63

**Strategy #4**

**Draw pictures/  
Create mental images/  
Foster visualization**

64

**The power of models and representations**

**Siti packs her clothes into a suitcase and it weighs 29 kg.**

**Rahim packs his clothes into an identical suitcase and it weighs 11 kg.**

**Siti's clothes are three times as heavy as Rahim's.**

**What is the mass of Rahim's clothes?**

**What is the mass of the suitcase?**

65

**The old (only) way:**

Let S = the weight of Siti's clothes

Let R = the weight of Rahim's clothes

Let X = the weight of the suitcase

$$S = 3R \quad S + X = 29 \quad R + X = 11$$

so by substitution:  $3R + X = 29$

and by subtraction:  $2R = 18$

so  $R = 9$  and  $X = 2$

66

Or using a model:

|       |  |  |  |  |
|-------|--|--|--|--|
| 11 kg |  |  |  |  |
| Rahim |  |  |  |  |
|       |  |  |  |  |
| Siti  |  |  |  |  |
| 29 kg |  |  |  |  |

67

And next:

Look at the power of  
context

68

## My Store


SALE  
Pencils 3¢  
Pens 4¢  
Erasers 5¢

Limit of 3 of each!

SO?

69

## Your turn

Pencils 7¢  
Pens 8 ¢  
Erasers 9 

Limit of 10 of each.

I just spent 83 ¢ (no tax) in this store.  
What did I purchase?

70

## Single-digit number facts

- More important than ever, BUT:
  - facts with contexts;
  - facts with materials, even fingers;
  - facts through connections and families;
  - facts through strategies; and
  - facts in their right time.

71

## Deep dark secrets

- $7 \times 8$ , 5 6 7 8
- $9 \times 6$ , 54 56 54 since  $5+4=9$
- $8 + 9$  .....  $18 - 1$  no,  $16 + 1$
- $63 \div 7 =$      $7 \times \underline{\quad} = 63$

72

### Some data

|  |    |   |
|--|----|---|
|  |    |   |
|  |    |   |
|  |    |   |
|  |    |   |
|  | 40 | 4 |
|  | 10 | 2 |
|  | 30 | 4 |
|  |    |   |

73

### Predict some additional data

|  |    |   |
|--|----|---|
|  |    |   |
|  |    |   |
|  |    |   |
|  |    |   |
|  | 40 | 4 |
|  | 10 | 2 |
|  | 30 | 4 |
|  |    |   |

74

### All the numbers – so?

|  |    |   |
|--|----|---|
|  |    |   |
|  | 45 | 4 |
|  | 25 | 3 |
|  | 15 | 2 |
|  | 40 | 4 |
|  | 10 | 2 |
|  | 30 | 4 |
|  | 20 | 3 |

75

### A lot more information (where are you?)

|                |    |   |
|----------------|----|---|
|                |    |   |
| Roller Coaster | 45 | 4 |
| Ferris Wheel   | 25 | 3 |
| Bumper Cars    | 15 | 2 |
| Rocket Ride    | 40 | 4 |
| Merry-go-Round | 10 | 2 |
| Water Slide    | 30 | 4 |
| Fun House      | 20 | 3 |

76

### Fill in the blanks

| Ride           | ??? | ??? |
|----------------|-----|-----|
| Roller Coaster | 45  | 4   |
| Ferris Wheel   | 25  | 3   |
| Bumper Cars    | 15  | 2   |
| Rocket Ride    | 40  | 4   |
| Merry-go-Round | 10  | 2   |
| Water Slide    | 30  | 4   |
| Fun House      | 20  | 3   |

77

At this point,  
it's almost anticlimactic!

78

## The amusement park – page 35

| Ride           | Time | Ticket |
|----------------|------|--------|
| Roller Coaster | 45   | 4      |
| Ferris Wheel   | 25   | 3      |
| Bumper Cars    | 15   | 2      |
| Rocket Ride    | 40   | 4      |
| Merry-go-Round | 10   | 2      |
| Water Slide    | 30   | 4      |
| Fun House      | 20   | 3      |

79

## Air and Space Museum (pg 40)

- Open 10 a.m. – 5:30 p.m
- Bus trip 2 ½ hours each way
- Bus pick-up at 8 a.m., must be back at 5 p.m.
- 2 movies and 4 free major exhibits
- Each movie costs \$3.25 and lasts 35 minutes.

SO?

80

## Pizza

**Small (6 slices)**  
**12" in diameter**  
**\$5.75**

**Large (8 slides)**  
**16" in diameter**  
**\$8.00**

SO?

81

## Dear sirs:

"I am in Mrs. Eaves Pre-algebra class at the Burn Middle School. We have been studying the area of shapes such as squares and circles. A girl in my class suggested that we compare the square and round pizzas sold by your store. So on April 16 Mrs. Eaves ordered one round and one square pizza from your store for us to measure, compare and...

82

## The search for sense-making/future leaders

"What is the reason for the difference in the price per square inch of these two pizzas? Is it harder to cook a round pizza? Does it take longer to cook? Because if 3.35 cents per square inch is acceptable for the square pizza, then the same price per square inch should be used for the round pizza, making the price \$10.31 instead of \$10.99.

Thanks for the tasty lesson in pizza values."  
 Sincerely,  
 Chris Collier

83

## Fruit Punch

- 2/3 cups cranberry juice
- 1 ½ cups grapefruit juice
- 1 1/3 cups orange juice
- 1 ¼ pints pineapple juice
- 2 ¼ quarts club soda

1 cup = 8 ounces  
 2 cups = 1 pint  
 2 pints = 1 quart  
 4 quarts = 1 gallon

84

### For Example:

$$F = 4 (S - 65) + 10$$

Find F when S = 81

Vs.

First I saw the blinking lights... then the officer informed me that:

The speeding fine here in Vermont is \$4 for every mile per hour over the 65 mph limit plus a \$10 handling fee.

85

$$\text{Connecticut: } F = 10 (S - 55) + 40$$

Maximum speeding fine: \$350

- Describe the fine in words
- At what speed does it no longer matter?
- At 80 mph how much better off would you be in VT than in CT?
- Use a graph to show this difference

86

### Strategy #5

Embed the mathematics in contexts;

Present the mathematics as problem situations.

87

### Implementing Strategy #5

Here's the math I need to teach.

When and where do normal human beings encounter this math?

88

Which class do YOU want to be in?

89

### Your turn

Using a fast food menu:

Given my grade \_\_\_\_\_ curriculum:

Statistics (graphs, averages)

Measurement (oz., gal, lbs)

Operations (when and why  $+/-/x/\div$ )

Geometry (area, volume, shape)

I could use the menu to foster understanding of what I am responsible for teaching by asking \_\_\_\_\_

90



## McDonalds Menu

What do they sell?

91

## McDonalds Ideas

- Order lunch
- Happy meal
- Sodas
- Dinner for 4 up to \$20
- French fries
- Surveys and graphs
- Combinations
- Other??

92

**Peter Dowdeswell of London,  
England holds the world record  
for pancake consumption!**

**62  
6" in diameter,  
3/8" thick pancakes,  
with butter and syrup  
in 6 minutes 58.5 seconds!  
SO?**

93

## So?

- About how high a stack? Show and explain
- Exactly how high?
- How fast?
- How much?
- Could it be, considering the size of the stomach?
- What's radius of single 3/8" thick pancake of same volume?
- Draw a graph of Peter's progress.

94

## Part 3

Tying things together:

**Skin**

95

## TIMSS Video Study 1

- Teacher instructs students in a concept or skill.
- Teacher solves example problems with class.
- Students practice on their own while the teacher assists.
- In other words.....

96

## Putting it all together one way

Good morning class.

Today's objective: Find the surface area of right circular cylinders.

Open to page 384-5.

Example 1:



$$S.A. = 2\pi rh + 2\pi r^2$$

Find the surface area.

Page 385 1-19 odd

97

## TIMSS Video Study 2

- Teacher presents complex, thought-provoking problem
- Students struggle with the problem individually and in groups
- Student present their work
- Teacher summarizes solutions and extracts important understandings
- Students work on a similar problem

98

## Putting it all together another way

Overheard in the ER as the sirens blare:

"Oh my, look at this next one. He's completely burned from head to toe."

"Not a problem, just order up 1000 square inches of skin from the graft bank."

You have two possible responses:

- Oh good – that will be enough.

OR

- Oh god – we're in trouble.

99

- Which response, "oh good" or "oh god" is more appropriate?
- Explain your thinking.
- Assuming you are the patient, how much skin would you hope they ordered up?
- Show how you arrived at your answer and be prepared to defend it to the class.

100

## Powerful Teaching

- Provides students with better access to the mathematics:
  - Context
  - Technology
  - Materials
  - Collaboration
- Enhances understanding of the mathematics:
  - Alternative approaches
  - Multiple representations
  - Effective questioning

101

## Last and most powerfully:

Make "why?"  
"how do you know?"  
"convince me"  
"explain that please"  
your classroom mantras

102

### **To recapitulate:**

1. Incorporate on-going cumulative review
2. Parallel literal to inferential to evaluative comprehension used in reading
3. Create a language-rich classroom
4. Draw pictures/create mental images
5. Embed the math in contexts/problems

And always ask them "why?"

103

## **Part 4**

### **Final thoughts on moving forward**

104

"Most teachers practice their craft behind closed doors, minimally aware of what their colleagues are doing, usually unobserved and under supported. Far too often, teachers' frames of reference are how they were taught, not how their colleagues are teaching. Common problems are too often solved individually rather than by seeking cooperative and collaborative solutions to shared concerns."

- Leinwand – "Sensible Mathematics"

105

### **Processing Questions**

- What are the two most significant things you've heard in this presentation?
- What is the one most troubling or confusing thing you've heard in this presentation?
- What are the two next steps you would support and work on to make necessary changes?

106

### **Finally – let's be honest:**

Sadly, there is no evidence that a day like today makes one iota of difference.  
You came, you sat, you were "taught".  
I entertained, I informed, I stimulated.  
But: It is most likely that your knowledge base has not grown, you won't change practice in any tangible way, and your students won't learn any more math.

107

### **Prove me wrong**

by  
Sharing  
Supporting  
Taking Risks

108

**Next steps: Sharing**  
**“Practice-based professional interaction”**

- Professional development/interaction that is situated in practice and built around “samples of authentic practice.”
- Professional development/interaction that employs materials taken from real classrooms and provide opportunities for critique, inquiry, and investigation.
- Professional development/interaction that focuses on the “work of teaching” and is drawn from:
  - mathematical tasks
  - episodes of teaching
  - illuminations of students’ thinking

109

**Next steps: Supporting**  
**The mindsets with which to start**

- We’re all in this together
- People can’t do what they can’t envision. People won’t do what they don’t understand. Therefore, colleagues help each other envision and understand.
- Can’t know it all – need differentiation and team-work
- Professional sharing is part of my job.
- Professional growth (admitting we need to grow) is a core aspect of being a professional

110

**Next steps: Taking Risks**  
**It all comes down to taking risks**

While “nothing ventured, nothing gained” is an apt aphorism for so much of life, “nothing risked, nothing failed” is a much more apt descriptor of what we do in school.

Follow in the footsteps of the heroes about whom we so proudly teach, and  
**TAKE SOME RISKS**

111

**Thank you.**

112

## **Selected Research Citations**

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